

Mathematical models for the determination of archaeological potential

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By following the practical approach adopted by the archaeological and geological team for the determination of archaeological potential, page rank models – currently used for classifying search engines web pages – appear to be suitable for determining the archaeological potential of the urban area of Pisa. The reason for this lies in the fact that the importance of a find is not absolute but basically depends on ‘nearby’ finds, which attribute importance to it: this is the same criterion implemented for page ranking. This report will briefly describe the classic page rank model and show how it may be applied to the determination of archaeological potential. Furthermore, we will also present a simpler model based on smoothing of the available data, which will be compared to the page rank-based model. The two different models will be tested via simulations.

Keywords: page rank, archaeological potential, Perron-Frobenius theorem, sparse matrices, smoothing

Introduction

Based on the discussions between the mathematical team and the archaeological and geological teams, an analogy arose between the criteria used for attributing archaeological potential and the criteria used for assigning importance to web pages in search engine algorithms.

We will try to make this analogy clear. When determining archaeological potential, geo-morphological and archaeological data are integrated in order to assign context, i.e. the traces of human actions or of natural events are gradually combined with a summary interpretation process (activities, groups of activities) and a period map of finds is defined that allows – through analytical work – the implementation of a map of archaeological potential for a certain historical period. Finally, the different historical periods are ‘overlapped’ to reach the final result. The interpretation process of the context consists in achieving a stratigraphic categorisation from a series of archaeological and geological data, based upon the *spatial and functional relations* among the various finds. This process becomes further evident when

determining the activities and group of activities, and also with the construction of the map of potential by archaeological period. For example, the erection of a *column* could represent a stratigraphic activity, which is deduced from the rests of the *column* itself, from the presence of a base or of nearby flooring. These activities, in turn, may result in the definition of a group of activities, such as a *domus*: a number of columns, together with floor structures, walls, windows, etc., which are adequately located in relation to each other, lead us to deduce the presence of a *domus*. More than one *domus*, in turn, together with dwellings and other structures (e.g. temples, roads or landfills, etc.) lead the archaeologist to infer the presence of an inhabited settlement.

The key issue of this analysis from an abstract viewpoint – leaving aside, therefore, the know-how and experience of the archaeologists and geologists – is the identification of the *relations* that exist among the various finds, both in spatial terms (i.e. the location in space) and in functional terms (i.e. which is or could be their function). In other words, the presence of a particular find near another that has already

been discovered could *strengthen or weaken* the probability that they will form a more complex structure, and so strengthen or weaken the archaeological potential of the area itself. This is exactly the criteria upon which page ranking algorithms are based, whereby each web page attributes importance to the web pages it points to (via a link) and, in turn, receives importance from the web pages it receives a link from.

The report is structured as follows: Section 2 will briefly describe the two models that are mainly used in literature for determining archaeological potential: the first is based on map algebra, the second on regression. Section 3 will present the page rank-based model. After describing the classic model, we will point out the changes that need to be made to adapt it to archaeological potential and we will present some simulations. Section 4 will introduce an alternative model for the determination of archaeological potential. This simpler model, based upon smoothing techniques, will include a number of simulations, in order to compare it with the page rank-based model. Finally, we will provide a comparison of the various models in section 5 and then present our final conclusions.

2. Existing models in literature

2.1 Deductive approach using map algebra

One of the approaches used in literature (probably the simplest) to predict archaeological potential consists of a predictive model capable of generating a decision rule. The input needed to determine this rule may be, for instance, land configuration (plain/slope), the presence of nearby water sources or soil type. These features may be combined into rules such as

$$(slope \geq 10) \cap (distance\ from\ source \geq 1\ km) \cup (soil \neq A)$$

in order to predict the presence or absence of an archaeological site. Variants to this approach may include assigning 'weights' to the different conditions, so that more importance is given to some conditions and less to others. At the same time, significance tests may be used to evaluate whether the proposed predictive model may be associated to the presence of archaeological sites within a certain confidence interval. The reader can see (CUMMING 1997; WHEATLEY 2002) for these kinds of models.

Models based on these rules are very easy to implement; however, they provide on/off results – for example, the presence or absence of an archaeological site – and do not go further than simply juxtaposing a number of easy rules. In other words, they do not exploit the power of a mathematical model or the computing capabilities of a computer.

2.2 Approach based on regression

Literature also provides another approach for de-

termining archaeological potential (WHEATLEY, 2002), based on the application of linear (or logistic) regression. This approach arises from the need to reply to questions that the above-described method cannot answer. For instance:

- How can a predictor influence the model?
- How can *continuous* quantities instead of discrete quantities be predicted?

Linear regression can be used to answer these questions, and is an easy approach, both in terms of its implementation and from a mathematical viewpoint. Without entering into details, all linear regressions produce equations of the following type:

$$y = a + b_1x_1 + \dots + b_kx_k,$$

where y is the variable that must be predicted (for example the archaeological potential), and x_1, \dots, x_k are the inputs of the linear regression. In

other words, by estimating coefficients a, b_1, \dots, b_k on the basis of the data available, a y value can be found, which will be used for making the prediction. Several variants may also be introduced in the regressive models (single or multiple regressions, non-linear regressions, statistical regressions, etc.). For further details on linear regression, also in relation to archaeology, please consult (SHENNAN 1997; WESCOTT 2000).

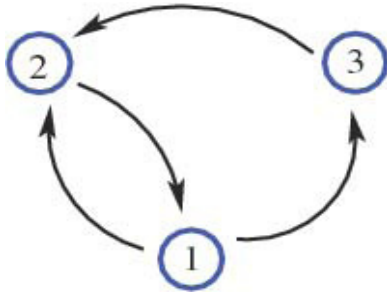
Although the approaches based on linear regression have the benefit of using variables to predict further variables, the model they implement is too simple and does not take into account the great complexity that must be considered when determining archaeological potential. This is so true that current models based on linear (or logistic) regression are often not preferred to those based on map algebra.

3. Determination of archaeological potential based on page ranking

3.1 Standard page-rank model

This paragraph will describe the general ideas and details of the most common mathematical models used for attributing a value of importance to web pages regardless of the value of their content and *solely on the basis of the interconnections between the pages*. For greater details please refer to a (LANGVILLE 2006).

Let us assume that we have n web pages and that they are numbered with integers from 1 to n . It is useful to use a directed graph to describe the World-Wide Web, in which the nodes represent the pages available on the Web and the directed edges describe the connections of these pages. More specifically, an edge connects node i with node j if page i has a link that points to page j . For example, if our WWW is made of 3 pages in which page 1 points to page 2 and to page 3, page 2 points to page 1, and page 3 points to page 2, the graph would be as follows



A directed graph may be univocally described by an adjacency matrix $H = h_{ij}$ of size $n \times n$ in which $h_{ij} = 1$ if a directed edge connects node i with node j (if page i contains a link to page j); otherwise $h_{ij} = 0$. The adjacency matrix associated to the above graph is

$$H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

The criterion used to determine the importance of the web pages may be summarised as follows: a page i that points to other pages, for example j_1, \dots, j_k distributes its importance in equal parts to pages j_1, \dots, j_k , and therefore gives $1/k$ of its importance to the pages it points to.

$$d_i = \sum_{j=1}^n h_{ij}$$

In this model, if we call d_i the sum of the entries in the i -th row of H , assuming $d_i \neq 0$ per $i=1, \dots, n$, and if we indicate by w_j the importance of page j , the following holds

$$w_j = \sum_{i=1}^n w_i \frac{h_{ij}}{d_i}, \quad j = 1, \dots, n.$$

For instance, in the case of the graph in the figure we have

$$\begin{cases} w_1 = w_2 \\ w_2 = 1/2w_1 + w_3 \\ w_3 = 1/2w_1 \end{cases}$$

This is simply a problem of eigenvalues and eigenvectors formulated in the following manner. If

$$e = (1, 1, \dots, 1)^T, \quad d = He, \quad \text{and} \quad D = \text{diag}(d), \quad \text{then} \\ w^T M = w^T, \quad M = D^{-1}H,$$

where $w^T = (w_1, \dots, w_n)$. This naive formulation, however, leads to a series of technical problems, listed below:

1. What happens if $d_i = 0$ for some i ? This happens when pages do not point to anything. This is not an unusual problem, indeed, some pages, such as a postscript files, may not have any links. Nodes with these features are called dangling nodes.
2. Is there always a solution?
3. Is the solution unique (modulo scalar multiples)?
4. Is the solution positive?
5. How can it be computed?

It should be noted that dangling nodes are identified since they correspond to the lines of H having all zero entries. In order to address the problem of dangling nodes, a slight modification is made to the model. More precisely, the initial matrix of adjacency

H is replaced with a new matrix \hat{H} which coincides with H everywhere apart from the zero lines where

the entries of \hat{H} are all equal to $1/n$. From a modelling viewpoint, it is as if we were to impartially assume that a document that in the model does not quote any other document in the web, were to quote all the existing documents in the new modified model. Therefore, it evenly distributes $1/n$ of its importance to all

of them. Matrix \hat{H} is therefore expressed as

$$\hat{H} = H + ue^T$$

where u is the vector with entry 1 in correspondence with the dangling nodes and zero elsewhere. Further below, matrix $M = \hat{D}^{-1}\hat{H}$, where $\hat{D} = \text{diag}(\hat{d})$, $\hat{d} = \hat{H}e$ will be expressed as M .

The answer to question 2 is affirmative: $Me=e$ and, therefore, 1 is an eigenvalue; consequently, w is any left eigenvector corresponding to the eigenvalue 1 . To reply to the other questions, we must report some classic results of the Perron-Frobenius theory regarding non-negative matrices.

Theorem: Let A be a $n \times n$ matrix with non-negative entries. Then, an eigenvalue λ of A exists such that $\lambda = \rho(A) \geq 0$. Also, a right eigenvalue x and left eigenvalue y correspond to λ with non-negative entries. Furthermore, if A is irreducible then λ is simple and the eigenvalues x and y have positive entries. Finally, if A has positive entries, then λ is the only eigenvalue of maximum module.

According to the Perron-Frobenius theorem, each solution has non-negative entries. However, the non-negativity condition, on its own, does not guarantee a unique solution (modulo scalar multiples), whereas a condition of irreducibility does. It is easy to construct networks of interconnected pages that have a reducible adjacency matrix. Therefore, the model thus introduced is still not appropriate. In the case of

irreducible and non-negative matrices, other eigenvalues may exist that have the same module of the spectral radius. This leads to serious problems from an algorithmic viewpoint. In order address these problems, the matrix M is replaced with the following matrix

$$A = \gamma M + (1 - \gamma)ev^T, \quad 0 < \gamma < 1,$$

where v is an arbitrary vector with negative entries such that $v^T e = 1$, called personalisation vector, and γ is a parameter, $\gamma = 0.85$ is usually chosen. Thus, matrix A has positive entries. The solution exists, therefore; it is unique, and $\rho(A)$ is the only eigenvalue of module 1. From a modelling viewpoint, it is as if the importance of a page were divided into two parts: a γ fraction is distributed on the basis of the links as in the original model, and a $1 - \gamma$ complementary fraction is distributed to all the other pages according to a criterion resulting from vector v . If, for example, $v = (1/n)e$, distribution shall be uniform to all Web pages.

3.2 Using the page rank model for determining archaeological potential

In order to apply a page rank model to the determination of archaeological potential, we will divide the subsurface of the urban area of Pisa into $m \times n \times p$ three-dimensional cells where the first two coordinates i, j define the items of a horizontal section of the land and the third coordinate k defines the deposit taken into consideration. The aim, therefore, is to assign an archaeological potential to each cell (i, j, k) ,

expressed by a real, non-negative number x_{ijk} , by $i = 1, \dots, m$, $j = 1, \dots, n$, $k = 1, \dots, p$. One of the conditions that we would like the solution of our model to meet is that

$$x_{ijk_1} \leq x_{ijk_2} \quad \text{se } k_1 \leq k_2$$

The explanation for this property lies in the fact that the archaeological potential of each cell (i, j, k) should be more appropriately interpreted as the potential obtained when *digging vertically from the surface down to that cell*. For this reason, as the excavation goes deeper, the archaeological potential increases.

The main change that needs to be made to the page rank model in order to adapt it to the determination of archaeological potential regards the criterion for defining the 'closeness' between cells, i.e. for defining the influence that every single cell has on the other cells. In the classic page rank model, the area of influence of a web page is defined by the links leaving the page itself. When determining the criteria of influence of the cells, therefore, we will bear in mind the following considerations, arising from detailed

discussions with the archaeological and geological teams.

- Building a page rank model in which each node of the graph, which corresponds to a cell of coordinates (i, j, k) , is identified by an integer r such that $1 \leq r \leq mnp$. The number r is determined via lexicographic;
- Creating a matrix $N \times N$ with $N = mnp$, $H = (h_{r,s})$ such that $h_{r,s}$ is the part of importance that cell r transfers to cell s . The value of $h_{r,s}$ is between 0 and 1;
- Using the archaeological information available (if available) for each cell in a twofold manner: on the one hand, in a *relative manner*, to construct the entries of matrix H that controls the transfer of importance of the cells, and on the other hand, in an *absolute manner*, giving importance to the specific cell that contains an archaeological find;
- The construction of the matrix that controls the transfer of importance is carried out on the basis of the categories used for classifying the archaeological finds. In particular, the category characterises both the 'geometry' and the values of distribution of importance. For example, a cell that contains finds relating to a road may interest contiguous cells located along a line;
- When building matrix H , the normalization condi-

tion $\sum_{j=1}^N h_{ij} = 1$ is maintained, which states that the entire importance of a cell is contained in the distribution: importance is neither amplified nor reduced;

• In the second part in which we wish to attribute a higher value d_k to one (or more) x_k entries of the archaeological potential, we may proceed as follows:

- o We may force the entry to assume the assigned value. In this case, the system $x^T H = x$ with condition of normalization $\sum x_i = 1$ and

condition $x_k = d_k$ becomes a linear, non-homogeneous and over-determined system, with more equations than unknowns. It must be treated, therefore, with least-squares techniques;

- o A homogeneous approach may be maintained by renouncing matrix normalization (stochasticity), i.e. scaling the lines of H with the assigned factor. In this case, matrix H is replaced by $A = DH$,

$D = \text{diag}(d_1, \dots, d_N)$ and the problem calculates the Perron vector of A , i.e. the non-negative

vector x such that $x^T A = \rho x^T$, where $\rho > 0$ is the spectral radius of A . Indeed, since it is no longer stochastic, matrix A may not necessarily have spectral radius 1. According to this appro-

ach, by scaling the rows, we give different weight to the importance that each cell distributes to the others. It should be observed that the condition

$$\sum_{i=1}^N d_{ii} = 1, d_{ii} \geq 0$$

according to which $\sum_{i=1}^N d_{ii} = 1, d_{ii} \geq 0$, does not guarantee that the spectral radius of A remains 1;

o A homogeneous approach may also be maintained by scaling the columns of H with the assigned factor. In this case, matrix H is replaced by $A=HD$. According to this approach, the importance that each cell receives is scaled (amplified or compressed);

- Using geological information in a binary manner, i.e. considering or excluding cells. Alternatively, using a weight between 0 and 1 in a multiplicative manner;
- The condition according to which archaeological potential increases as digging deeper into the ground may be implemented in at least two ways:

o by including links in the graph that connect cell (i,j,k) with cell $(i,j,k+1)$; thus, the deep cells receive more importance than the surface cells;

o adopting the quantities $y_{ijk} = \sum_{h=1}^k x_{ijh}$, where

x_{ijh} is the Perron vector of H , as a solution for archaeological potential.

3.3 Simulations

Future work to be conducted with the archaeologists, after categorisation of the finds, will consist in assigning the distribution and the geometry of the importance values that a find in a cell attributes to the other cells. Meanwhile, this paragraph will describe several versions of the page rank algorithm adapted to the determination of archaeological potential. These algorithms will be implemented in a simplified version in order to conduct simulations.

For this reason, we will assume that the model is one-dimensional in this report. After numbering the cells of the three-dimensional problem from 1 to N ,

let us define matrix $A = a_{ij}$ of size N such that a_{ij} is the part of archaeological potential that cell j gives to cell i . The entries of A are non-negative and such

that $\sum_j a_{ij} = 1$, i.e. matrix A is stochastic. Matrix A is defined by the information available. The problem to be solved, in its non-homogenous formulation, therefore, is

$$\begin{aligned} Ax &= x \\ x_j &= b_j, \quad j \in \Delta \end{aligned}$$

where Δ is the set of indices of the cells of which

we have archaeological potential information and b_j are the values of this potential. Considering its homogeneous formulation, instead, the problem assumes one of the two following expressions

$$DAx = \lambda x, \quad ADx = \lambda x$$

where λ is the eigenvalue of maximum module of DA and AD , respectively, whereas D is the diagonal matrix whose diagonal entries give weight to the archaeological potential of the corresponding cell.

Therefore, the non-homogeneous problem is formed by an over-determined system that needs to be solved, for example, with the least-squares technique. If

we replace the known values of x_j , it becomes a linear system in N equations and $N-d$ unknowns, where d is the cardinality of Δ . Instead, the homogeneous model is the solution of an eigenvalue problem. If there is no archaeological information, the initial matrix of

the weights is assumed as being equal to $1/N \cdot e^T$, i.e., the matrix of entries all equal to $1/N$. The reason for this is that, since there is no information, each entry gives importance (very negligible since equals $1/N$) to all the others. This is a more favourable hypothesis than assuming the identity matrix as matrix of the weights (which would seem more natural), in which each item gives importance only to itself. The advantage in choosing the matrix of weights with all entries equal to $1/N$ lies in the fact that the matrix is positive and therefore the theorem of Perron-Frobenius is valid, which guarantees the existence and uniqueness of solution x . Moreover, since information is missing, x is the vector of entries all equal to $1/N$ (if we normalize the sum to 1). It is therefore reasonable, from a modelling viewpoint, for all cells to have archaeological potential equal to each other, thus negligible. If we had started from an identical matrix, the solution would not have been unique and the problem would have been inconsistent.

When archaeological information is available, we proceed along two levels, as already previously described. First, we form matrix A by assigning the value of weights with which each cell j distributes its own importance to cell i . Then, we assign a value of importance to the find itself via multiplication by a diagonal matrix D which contains, for every entry of the diagonal, the value of importance associated to the find in the corresponding cell. Multiplication may be on the right or on the left of matrix A , depending on whether we wish to unload the weight of this value on the connections that leave the cells or are received by them. Finally, the Perron eigenvalue is found, associated to the matrix of weights multiplied (on the right or on the left) by the matrix of values of importance.

We chose $n=100$ for this simulation and entered finds in cells 15, 37, 39, 68, with importance, respectively, of 3, 1.5, 1.7, 2. With regard to distribution of the

weights, we simulated the following case

- the find of cell 15 gives importance to cells 3, 4, 5, 6, 7, 8 equal to 1/6;
- the find of cell 37 gives importance to cells 45, 47, 49, 51, 53, 55, 57, 59 equal to 1/8;
- the find of cell 39 gives importance to cells 46, 48, 50, 52, 54, 56, 58, 61 equal to 1/8;
- the find of cell 68 gives importance to cells 13, 14, 15, 16, 25 equal to 1/5;

By solving the non-homogeneous and the homogeneous problems, the values of archaeological potential are obtained, as in figure 1, where we have reported the solutions obtained for the stochastic matrix (i.e. without considering the importance of the finds, the non-homogeneous problem) and for the left-multiplication and right-multiplication for the diagonal matrix with the values of importance of the finds. With regard to the stochastic matrix, the distribution of archaeological potential resulting from the simulation is basically distributed according to the weights assigned to each cell.

Instead, the situation changes significantly if we assign importance to the finds. If we consider left-scaling – i.e. the case in which the importance of the find is ‘loaded’ on the weights received – a general increase in potential may be seen, which is higher in the areas where the finds have been discovered: the difference between the areas with greater archaeological potential and areas with lower archaeological potential is much more pronounced than in the stochastic case. With reference to the second case, archaeological potential is higher in cells 13, 14, 15, 16, 25, which receive importance in a more ‘concentrated’ manner from cell 68, which distributes a value of importance equal to 2. If we consider right-scaling – i.e. the case in which the importance of the find is ‘loaded’ on the weights sent – the iteration between the weights and the importance of the finds appears to be more significant when assigning final archaeological potential. Indeed, with regard to this third case, archaeological potential is higher in cell 68, which besides having a value of importance of the find equal to 2, distributes its weight in a more ‘concentrated’ manner with respect to the other finds.

4. Alternative model

This section will describe an alternative model for the determination of archaeological potential starting from data available from the excavations and from already existing finds. Our aim is to provide a ‘basic model’ that is simpler than the page rank-based model and which can be used to test the goodness of the page rank-based method.

This alternative model is not built upon considerations regarding archaeological practice but consists of a ‘simple’ approximation of existing data: it is based on the principle according to which archaeologi-

cal potential tends to increase by proceeding from a point where it is lower to a point where it is higher, and vice versa. We used the matlab csaps function, which provides an approximation of data by means of cubic splines.

We chose $n=100$ for the simulation and we included finds in cells 15, 37, 39, 68, with importance, respectively, of 3/10, 1.5/10, 1.7/10, 2/10. In order to facilitate comparison between the two different methods, the cells and the relative importance of the finds were the same used for simulation with the page rank-based models. The solution of the problem given by the approximation of the data provided is shown in Figure 2.

Since the importance of the find is the only condition that counts in this case, smoothing simply constructs a curve adapting it to the points in which the values (of importance) are known. Actual smoothing was used for this simulation (this possibility can be modulated by means of a Matlab csaps function parameter), not interpolation, in order to assume the values of importance of the find (which will be assigned by the archaeological-geological team) and which may be subject to an error.

5. Comparison between models and conclusions

In this final section, we will compare the different models used, those based on the page rank model and those on smoothing. This comparison will be carried out since we are convinced that the models existing in literature – based on map algebra or regression – are not appropriate for determining archaeological potential as envisaged for the MAPPA project, for the reasons already mentioned in the section regarding existing literature. In order to compare the various models we will refer to figure 3, which reports the simulations carried out in the previous sections.

The first main difference between the smoothing-based model and page-rank based model consists in the possibility to ‘distribute’ the importance of a cell to other cells, which is possible only in the page rank-based model. As already pointed out, this allows us to assign a certain ‘probability of importance’ in cells where there have been no finds. This is not possible in the smoothing-based process – nor in the models quoted in literature such as the map algebra or linear regression models. Indeed, if we observe the figure, it is possible to notice how the smoothing-based model (green curve) concentrates the archaeological potential only in the areas surrounding the finds, whereas the page rank-based models concentrate the archaeological potential also around the cells which acquire importance from the cells containing the finds.

Another important difference regards the essentially relative nature of the page rank-based model, compared to the essentially absolute nature of the smoothing-based model. In the page rank-based model, in addition to the weights – which represent the por-

tion of importance that each cell distributes to the other cells – we assigned an importance to each find by attributing a value to it which actually represented importance. This value, however, is not absolute but relative with respect to the values assigned to the other finds. Instead, in the smoothing-based model, the first problem to be addressed is the assignment of an absolute value for the potential of each cell, where known. Besides representing an additional evaluation to be carried out compared to the page rank-based model, it is also not particularly correct in theoretical and/or general terms. In fact, archaeological potential – together with characteristics that contribute to its determination (such as density of finds, rarity of finds) – is per se related to the area under examination and to the area's size.

5.1 Conclusions

In conclusion, we have presented two different models for estimating archaeological potential starting from already-discovered finds. We believe that existing models in literature, based mainly on map algebra and regression, are not suitable due to their extreme simplicity.

The preliminary work to be carried out in order to test these models on real data will consist in associating a 'distribution of importance' to each category of finds indicated by the archaeological team: the 'distribution of importance' will contain the informa-

tion that is provided by each find about what could be discovered nearby. Finally, geological information – which will be considered in a binary manner – was not included in the simulations since implementation of this information at modelling level is immediate.

We will focus our attention on the page rank-based model because this model allows us to take into consideration a series of characteristics that follow the practical methods used by archaeologists for determining archaeological potential. The smoothing-based model, instead, which only considers finds without 'distributing' their importance, will be used for comparison purposes. We have already pointed out the advantages of the page-rank based models but we believe that they are much greater than initially highlighted in this phase. For example, the page rank model will be implemented considering firstly the finds in a single period and then 'summing' all the various periods, and bearing in mind the different interactions that finds belonging to diverse periods may have. We must also consider that the 'distribution' of importance may be at the level of cells but also of objects (groups of cells), i.e. at various levels of complexity.

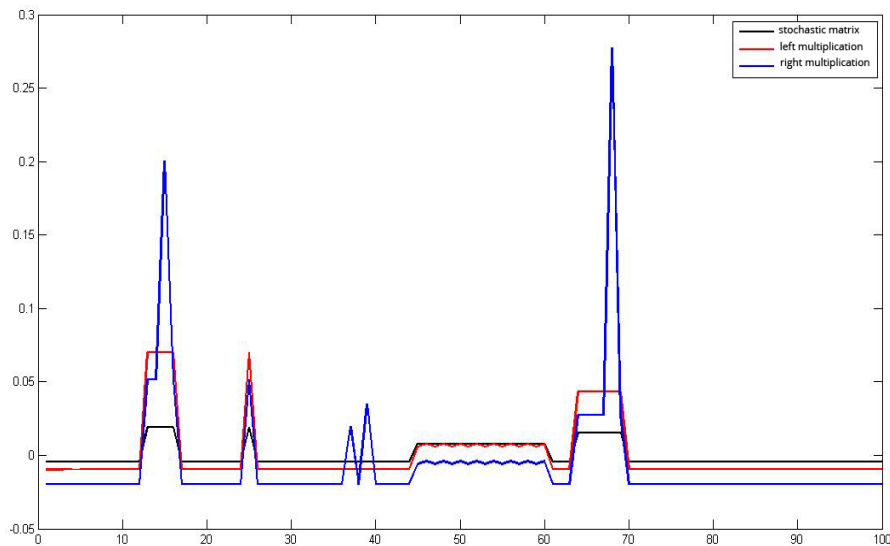


Fig.1 Estimates of archaeological potential as obtained from page rank-based models.

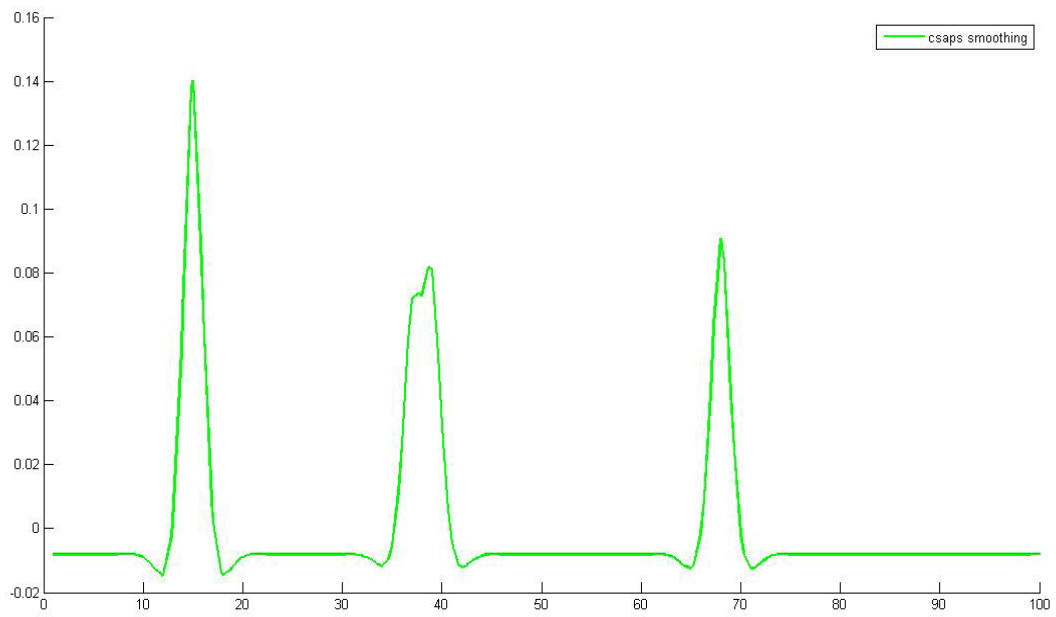


Fig.2 Estimate of archaeological potential obtained with smoothing and implemented by the matlab csaps function.

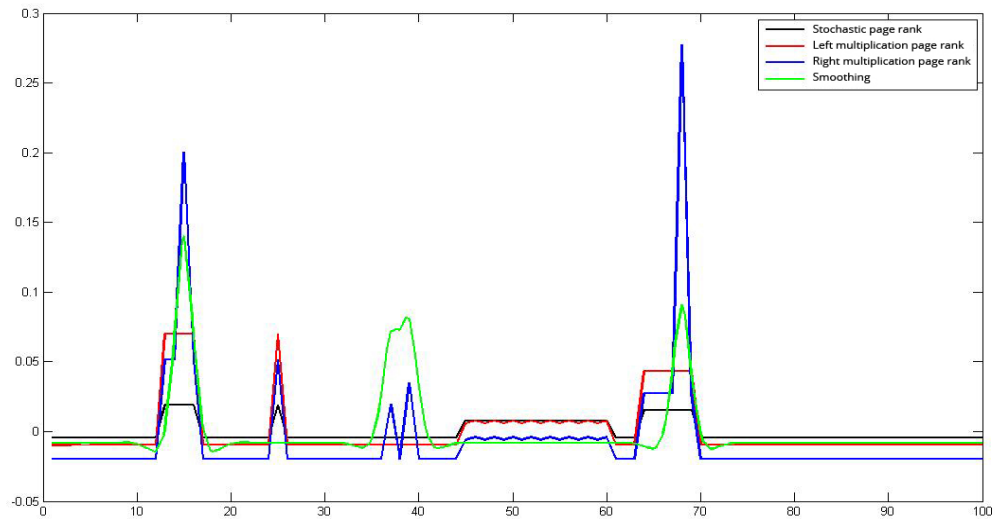


Fig.3 Estimate of archaeological potential obtained with all four models implemented

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